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Bounds on M_W, M_t, $\sin^2\theta_{\rm eff}^{\rm lept}$

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Abstract. Assuming that the standard model is correct and taking into account the lower bound on M_H from direct searches, we discuss bounds on M_W, $M_{\rm top}$, and $\sin^2\theta_{\rm eff}^{\rm lept}$ at various confidence levels. This permits us to identify theoretically favored ranges for these important parameters in the standard model framework, regardless of other observables. As an illustration of possible future developments, a hypothetical benchmark scenario, involving shifts $\lesssim 1\sigma$ in the experimental central values, is discussed.

1 Introduction

The general consensus at present is that the standard model (SM) gives a very good description of a multitude of phenomena from atomic energies up to the electroweak scale. On the other hand, a fundamental pillar of the theory, the Higgs boson, has not been found so far and some experimental observables put sharp constraints on its mass. This is particularly true of the M_W measurement. For instance, it was pointed out in [1] that the 2002 average value $(M_W)_{\rm exp} = 80.451 \pm 0.033\,{\rm GeV}$, in conjunction with $(M_t)_{\rm exp} = 174.3 \pm 5.1\,{\rm GeV}$, $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, $\alpha_{\rm s}(M_Z) = 0.118 \pm 0.002$, led to the prediction $M_H = 23^{+49}_{-23}\,{\rm GeV}$, and the 95% C.L. upper bound $M_H^{95} = 122\,{\rm GeV}$. The first value is embarrassingly low relative to the 95% C.L. lower bound $(M_H)_{\rm L.B.} = 114.4\,{\rm GeV}$ from direct searches [2], while the second one is only slightly larger. Since then the situation has changed significantly: repeating this analysis with the new experimental value $(M_W)_{\rm exp} = 80.426 \pm 0.034\,{\rm GeV}$ [2], we find, on the basis of the simple formulae of [1], the predictions $M_H = 45^{+69}_{-36}\,{\rm GeV}$, $M_H^{95} = 184\,{\rm GeV}$, which are much less restrictive. The predictions are further relaxed if one uses as inputs both $(M_W)_{\rm exp}$ and the current average value $(s^2_{\rm eff})_{\rm exp} = 0.23150 \pm 0.00016$ [2], where $s^2_{\rm eff}$ is an abbreviation for $\sin^2\theta_{\rm eff}^{\rm lept}$. This analysis leads to

$$M_H = 112^{+69}_{-45}\,{\rm GeV}; \quad M_H^{95} = 243\,{\rm GeV}, \tag{1}$$

which are not far from the values currently derived from the global fit: $M_H = 96^{+60}_{-38}\,{\rm GeV}$, $M_H^{95} = 219\,{\rm GeV}$ [2].

There are three factors that single out the M_W determination as particularly significant:
(i) as illustrated in the above remarks, it places sharp restrictions on M_H;
(ii) The LEP2 and collider measurements of M_W are in excellent agreement with $\chi^2/{\rm Dof} = 0.3/1$;
(iii) the relevant electroweak correction Δr [3] has been fully evaluated at the two-loop level [4], an important theoretical achievement.

The aim of this paper is to derive bounds on M_W, $M_{\rm top}$, and $\sin^2\theta_{\rm eff}^{\rm lept}$ in the SM framework by comparing the experimental measurements of these three basic parameters at various confidence levels with the theoretical functions $M_W = M_W(M_H, M_t)$ and $s^2_{\rm eff} = s^2_{\rm eff}(M_H, M_t)$, for fixed values of M_H. The lower bound $(M_H)_{\rm L.B.}$ restricts the available parameter space and this leads to bounds on M_W, $M_{\rm top}$, and $\sin^2\theta_{\rm eff}^{\rm lept}$ that are significantly sharper than those derived from the experimental measurements. Of course, this approach assumes the validity of the SM and makes use of the $(M_H)_{\rm L.B.}$. Thus, the derived bounds may be regarded as theoretically favored domains for these parameters in the SM framework, regardless of other observables. As such, they may suggest plausible ranges of variability in future, more precise experimental determinations.

In Sect. 2 we examine the bounds derived from the theoretical functions $M_W = M_W(M_H, M_t)$ using the simple formulae from [1], as well as the new theoretical expressions presented in [5]. In Sect. 3 we extend the analysis to the functions $s^2_{\rm eff} = s^2_{\rm eff}(M_H, M_t)$ using the results of [1]. In Sect. 4 we present the conclusions and, as an illustration of possible future developments, we discuss a hypothetical benchmark scenario involving shifts of $\lesssim 1\sigma$ in the experimental central values. Appendix A discusses the effect on the analysis of the bounds due to the estimated errors in the $\Delta\alpha_h^{(5)}$ determinations, and Appendix B extends the anal-

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ysis of Sect. 4 to the case of a "theory driven" calculation of $\Delta\alpha_h^{(5)}$.

2 M_W and M_t

In this section we compare the experimental values of M_W and M_t with the theoretical SM curves $M_W = M_W(M_H, M_t)$ for fixed values of M_H, taking into account the lower bound $(M_H)_{\rm L.B.}$ on M_H from the direct searches. To simplify the analysis we take the restriction $M_H \geq 114.4\,$GeV to be a sharp cutoff rather than a 95% C.L. bound. The theoretical curves depend also on $\Delta\alpha_h^{(5)}$, the contribution from the first five quark flavors to the running of α at the M_Z scale. We use as inputs $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$ [6] and the "theory driven" calculation $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$ [7]. We also employ as inputs α, $G_{\rm F}$, M_Z, and $\alpha_{\rm s}(M_Z) = 0.118 \pm 0.002$ [1,2].

Figure 1 shows the theoretical SM curves $M_W(M_H, M_t)$ for $M_H = 114.4$ (dashed line), 139, 180, 224 GeV, $\Delta\alpha_h^{(5)} = 0.02761$, and $\alpha_{\rm s}(M_Z) = 0.118$, evaluated with the simple formulae of [1] in the effective scheme of renormalization [1,8], as well as the 68%, 80%, 90%, 95% C.L. contours derived from the current experimental values $(M_W)_{\rm exp} = 80.426 \pm 0.034\,$GeV, $(M_t)_{\rm exp} = 174.3 \pm 5.1\,$GeV. An interesting feature is that the theoretical curves are nearly linear over the range of M_t values considered. At a given C.L. the allowed region lies within the corresponding ellipse and below the $M_H = 114.4\,$GeV SM theoretical curve (dashed line), which we call the boundary curve (B.C.). As shown in Fig. 1, the B.C. barely misses intersecting the 68% C.L. ellipse, so that strictly speaking this region is not allowed when the $(M_H)_{\rm L.B.}$ restriction is imposed. It turns out that, to a good approximation, the maximum and minimum M_W and M_t values in a given allowed region are determined by the intersections of the B.C. with the associated ellipse. This interesting feature can be understood by a glance at Fig. 1. The allowed M_W and M_t ranges determined by such intersections are shown in Table 1 for the 80%, 90%, 95% C.L. domains. As M_H increases beyond 114.4 GeV, the allowed ranges decrease in size. At a given C.L. domain, the maximum allowed M_H corresponds to the theoretical curve $M_W(M_H, M_t)$ that just touches the associated ellipse. From Fig. 1 we can see that these values are $M_H \approx 139\,$GeV, 180 GeV, 224 GeV corresponding to the 80%, 90%, 95% C.L. domains.

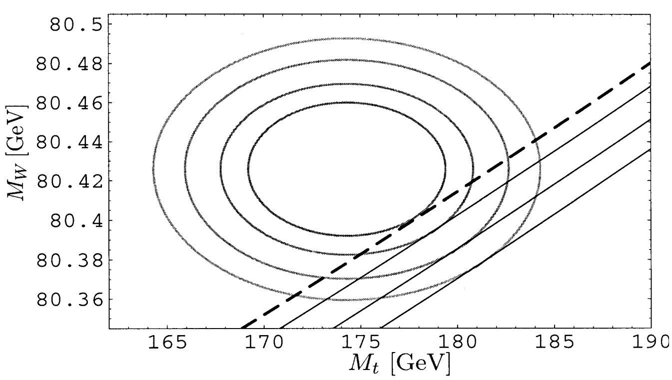

Fig. 1. 68%, 80%, 90%, 95% C.L. domains derived from $(M_W)_{\rm exp} = 80.426 \pm 0.034\,$GeV and $(M_t)_{\rm exp} = 174.3 \pm 5.1\,$GeV, together with the SM theoretical curves $M_W(M_H, M_t)$ for $M_H = 114.4$ (dashed line), 139, 180, 224 GeV from [1] with $\Delta\alpha_h^{(5)} = 0.02761$. At a given C.L. the allowed region lies within the corresponding ellipse and below the dashed boundary curve B.C.

In the above analysis $\Delta\alpha_h^{(5)}$ has been kept fixed at the central value $\Delta\alpha_h^{(5)} = 0.02761$. If it is allowed to vary according to $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, the analysis is somewhat more involved (see Appendix A). However, the conclusion is that the M_W, M_t ranges reported in Table 1 are at most affected by minor shifts.

Table 2 presents the M_W, M_t ranges evaluated with $\Delta\alpha_h^{(5)} = 0.02747$. In this case we see that there is a very narrow window of compatibility with the 68% C.L. domain. Otherwise, the M_W, M_t ranges are very similar to those in Table 1. It is interesting to note that compatibility with the SM improves as $\Delta\alpha_h^{(5)}$ decreases.

Tables 3 and 4 repeat the analysis of Tables 1 and 2 on the basis of the SM theoretical formulae presented in [5], which are based on a complete two-loop calculation of Δr [4] in the on-shell scheme of renormalization [3,9]. These tables employ $\Delta\alpha_h^{(5)} = 0.02761$ and $\Delta\alpha_h^{(5)} = 0.02747$, respectively. Again the 68% C.L. domain is not compatible with the SM curves subject to the $(M_H)_{\rm L.B.}$ restriction. The allowed M_W, M_t ranges in the 80%, 90%, 95% C.L. domains are similar but somewhat more restrictive than in

Table 1. Comparison of the experimental values $(M_W)_{\rm exp} = 80.426 \pm 0.034\,$GeV and $(M_t)_{\rm exp} = 174.3 \pm 5.1\,$GeV, at various C.L., with SM theoretical expressions based on [1] and $\Delta\alpha_h^{(5)} = 0.02761$. The table shows the ranges for M_W and M_t that, according to the SM, are compatible with the restriction $M_H \geq 114.4\,$GeV. In order to belong to the allowed regions, pairs of M_W and M_t values from these intervals should be chosen so that they lie within the corresponding C.L. domains. Within each C.L. domain, the M_W, M_t ranges decrease as M_H increases from 114.4 GeV

EFF scheme $\Delta\alpha_h^{(5)} = 0.02761$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.383–80.419	175.0–180.7
90% C.L.	80.371–80.431	173.1–182.6
95% C.L.	80.362–80.441	171.6–184.1

Table 2. Same as in Table 1, for $\Delta\alpha_h^{(5)} = 0.02747$

EFF scheme $\Delta\alpha_h^{(5)} = 0.02747$	range M_W [GeV]	range M_t [GeV]
68% C.L.	80.396–80.408	176.7–178.7
80% C.L.	80.383–80.422	174.6–180.8
90% C.L.	80.371–80.434	172.7–182.6
95% C.L.	80.362–80.443	171.3–184.0

Table 3. Same as in Table 1, with SM theoretical expressions from [5]

Awramik et al. [5] $\Delta\alpha_h^{(5)} = 0.02761$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.385–80.409	176.3–180.3
90% C.L.	80.370–80.423	173.9–182.7
95% C.L.	80.361–80.433	172.3–184.2

Table 4. Same as in Table 3, for $\Delta\alpha_h^{(5)} = 0.02747$

Awramik et al. [5] $\Delta\alpha_h^{(5)} = 0.02747$	range M_W [GeV]	range M_t [GeV]
80% C.L.	80.384–80.413	175.7–180.5
90% C.L.	80.371–80.426	173.6–182.7
95% C.L.	80.361–80.435	172.0–184.2

Tables 1 and 2. In particular, although the minimum M_W values are nearly the same, the maximum M_W values are from 10 to 7 MeV smaller than in Tables 1 and 2.

In Table 5 we present the mid-points of the M_W and M_t ranges in Table 2 with variations that cover the full intervals. At a given C.L., these are compared with the domains in M_W and M_t derived from $(M_W)_{\rm exp} = 80.426 \pm 0.034$ GeV and $(M_t)_{\rm exp} = 174.3 \pm 5.1$ GeV. As expected, the SM allowed M_W, M_t ranges are significantly restricted. To very good approximation, the mid-points (80.402, 177.7) GeV are independent of the C.L. and are shifted from the experimental central values $(M_W)^c_{\rm exp}$ and $(M_t)^c_{\rm exp}$ by $\Delta M_W = -0.71\sigma_{M_W}$ and $\Delta M_t = +0.67\sigma_{M_t}$.

In the case of Tables 1, 3, and 4, to very good approximation the mid-points are also independent of the C.L. and are given by (80.401, 177.9) GeV, (80.397, 178.3) GeV, and (80.398, 178.1) GeV, respectively. The largest shifts occur for Table 3, where the M_W mid-point is $0.85\sigma_{M_W}$ below $(M_W)^c_{\rm exp}$ and the M_t mid-point is $0.78\sigma_{M_t}$ above $(M_t)^c_{\rm exp}$.

The theoretical formulae employed in this analysis are of course subject to errors associated with the truncation of the perturbative series and the QCD corrections. A relevant question is: what is the effect of these errors in the determination of the allowed M_W, M_t ranges? We give two specific examples concerning the 90% C.L. domains in Tables 1 and 3. Using the estimated theoretical errors discussed in [1], we find that the 90% C.L. ranges in Table 1 become (60 ± 5) MeV in M_W and $(9.5^{+0.8}_{-0.9})$ GeV in M_t. Using the estimated theoretical errors of [5], we find that the corresponding 90% C.L. intervals in Table 3 become (53^{+3}_{-4}) MeV in M_W and $(8.8^{+0.6}_{-0.8})$ GeV in M_t. Thus, the effect of the errors in the theoretical formulae is to change the size of the allowed intervals by less than 10%. We also note that these modifications are significantly smaller than the experimental errors of M_W and M_t expected at TeV-LHC (cf. Sect. 4).

3 $\sin^2\theta_{\rm eff}^{\rm lept}$ and M_t

On the experimental side, we consider two possibilities: the current world average $(\sin^2\theta_{\rm eff}^{\rm lept})_{\rm exp} = 0.23150 \pm 0.00016$ [2] and the average derived from the leptonic observables $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)} = 0.23113 \pm 0.00021$ [2]. The difference between these values reflects the well-known dichotomy between the leptonic and hadronic determinations, which differ by $\approx 3\sigma$. On the theoretical side, the relevant electroweak correction is $\Delta r_{\rm eff}$ [8,10,11]. Unlike Δr, it has not been fully evaluated at the two-loop level. For this reason, we simply employ the formulae of [1] in the effective scheme of renormalization. They contain two-loop electroweak effects enhanced by powers $(M_t^2/M_W^2)^n$ $(n = 1, 2)$, as well as QCD corrections. For M_t, $\Delta\alpha_h^{(5)}$, and $\alpha_s(M_Z)$ we employ the same inputs as in Sect. 2.

Figure 2 shows the 68%, 80%, 90%, and 95% C.L. domains derived from the world average $(\sin^2\theta_{\rm eff}^{\rm lept})_{\rm exp} =$

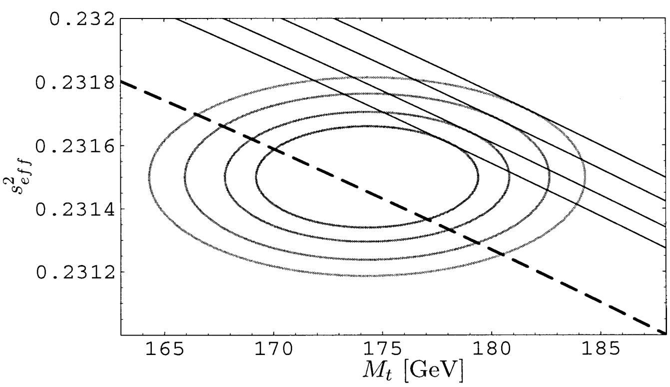

Fig. 2. 68%, 80%, 90%, 95% C.L. domains derived from $\sin^2\theta_{\rm eff}^{\rm lept} = 0.23150 \pm 0.00016$ and $M_t = 174.3 \pm 5.1$ GeV, together with the SM theoretical curves $s^2_{\rm eff}(M_H, M_t)$ for $M_H = 114.4$ (dashed line), 193, 218, 253, 289 GeV from [1] and $\Delta\alpha_h^{(5)} = 0.02761$. The allowed regions lie above the dashed B.C.

Table 5. Allowed M_W, M_t ranges from Table 2, expressed as mid-points and variations covering the corresponding intervals. They are compared with the ranges extracted from the experimental values

C.L.	$(M_W)_{\rm exp}$ [GeV]	Allowed M_W [GeV]	Allowed M_t [GeV]	$(M_t)_{\rm exp}$ [GeV]
68% C.L.	80.426 ± 0.034	80.402 ± 0.006	177.7 ± 1.0	174.3 ± 5.1
80% C.L.	80.426 ± 0.044	80.402 ± 0.020	177.7 ± 3.1	174.3 ± 6.5
90% C.L.	80.426 ± 0.056	80.402 ± 0.032	177.7 ± 5.0	174.3 ± 8.4
95% C.L.	80.426 ± 0.067	80.402 ± 0.041	177.7 ± 6.4	174.3 ± 10.0

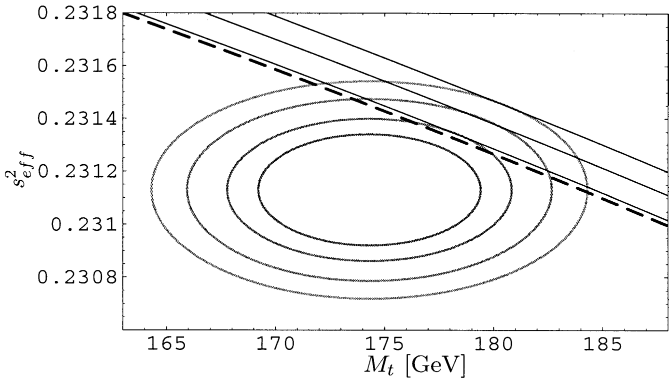

Fig. 3. 68%, 80%, 90%, 95% C.L. domains derived from $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)} = 0.23113 \pm 0.00021$ and $M_t = 174.3 \pm 5.1\,$GeV, together with the SM theoretical curves $s^2_{\rm eff}(M_H, M_t)$ for $M_H = 114.4$ (dashed line), 119, 143, 169 GeV from [1] and $\Delta\alpha_h^{(5)} = 0.02761$. The allowed regions lie above the dashed B.C.

0.23150 ± 0.00016 and $(M_t)_{\rm exp} = 174.3 \pm 5.1\,$GeV, as well as the SM theoretical curves $s^2_{\rm eff}(M_H, M_t)$ for $M_H =$ 114.4 (dashed line), 193, 218, 253, 289 GeV, evaluated with $\Delta\alpha_h^{(5)} = 0.02761$ and $\alpha_s(M_Z) = 0.118$. At a given C.L. the allowed region lies within the corresponding ellipse and above the B.C. Since in this case the center of the ellipses lies in the allowed regions, the situation is very different from that in Fig. 1. In fact, the reduction in parameter space is much less radical than in the (M_W, M_t) analysis. In particular, as shown in Fig. 2, the maximum $s^2_{\rm eff}$ and M_t values are not affected by the $(M_H)_{\rm L.B.}$ restriction and the minimum values are increased by relatively small amounts.

In the case of the leptonic average $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)} =$ 0.23113 ± 0.00021, the situation is depicted in Fig. 3. The allowed regions lie again within the C.L. ellipses and above the B.C. (dashed line). The 68% C.L. domain is clearly forbidden.

In the $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)}$, M_t analysis, the effect of varying $\Delta\alpha_h^{(5)}$ according to $\Delta\alpha_h^{(5)} = (\Delta\alpha_h^{(5)})^c \pm \sigma_{\Delta\alpha}$ is more pronounced than in the M_W, M_t case and, accordingly, we have derived the allowed intervals using the χ^2 analysis discussed in Appendix A. They are shown in Tables 6 and 7, for $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$ and $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$, respectively. To good approximation, the mid-points are again independent of the C.L. and are given by (0.23129; 177.3 GeV) in Table 6 and by (0.23129; 177.4 GeV) in Table 7. For a latter application in Sect. 4, we list also the mid-points of the $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)}$, M_t ranges defined by the intersections of the $\Delta\alpha_h^{(5)} = 0.02761$ and $\Delta\alpha_h^{(5)} =$ 0.02747 B.C. with the C.L. ellipses. They are (0.23133; 178.1 GeV) and (0.23130; 177.5 GeV), respectively. In the $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$ case, they differ by relatively small amounts from the mid-points in Table 6, mainly because the latter take into account the effect of the $\Delta\alpha_h^{(5)}$ variation discussed before. In the $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$ case, because of the smallness of the error, the effect is less significant and they almost coincide with the mid-points in Table 7. The shifts of the B.C. mid-points (0.23133; 178.1 GeV) and (0.23130; 177.5 GeV) from the experimental central values amount to $(+0.95\,\sigma_{s^2_{\rm eff}}, +0.75\,\sigma_{M_t})$ and $(+0.81\,\sigma_{s^2_{\rm eff}}, +0.63\,\sigma_{M_t})$, respectively.

In analogy with the discussion at the end of Sect. 2, we now study the effect of the theoretical errors on the determination of the $(\sin^2\theta_{\rm eff}^{\rm lept})_{(l)}$, M_t ranges. We consider, as an illustration, the 90% C.L. intervals in Table 7. Taking into account the theoretical errors discussed in [1], we find that these ranges become 0.00033 ± 0.00005 in $(s^2_{\rm eff})_l$ and $(10.5^{+1.0}_{-1.9})$ GeV in M_t. Thus, we see that the effect of the theoretical errors is more pronounced than in the M_W, M_t case, changing the size of the allowed domains by less than 15% in $(s^2_{\rm eff})_l$ and 18% in M_t.

Table 6. Comparison of the experimental values $(s^2_{\rm eff})_l =$ 0.23113 ± 0.00021 and $M_t = 174.3 \pm 5.1\,$GeV at various C.L. with SM theoretical expressions from [1] and $\Delta\alpha_h^{(5)} =$ 0.02761 ± 0.00036. The table shows the ranges for $s^2_{\rm eff}$ and M_t that, according to the SM, are compatible with the restriction $M_H \geq 114.4\,$GeV. In order to belong to the allowed regions, pairs of $s^2_{\rm eff}$ and M_t values from these intervals should be chosen so that they lie within the corresponding C.L. domains

EFF scheme $\Delta\alpha_h^{(5)} = 0.02761$	range $\left(\sin^2\theta_{\rm eff}^{\rm lept}\right)_l$	range M_t [GeV]
80% C.L.	0.23119–0.23139	174.7–179.9
90% C.L.	0.23111–0.23147	172.1–182.6
95% C.L.	0.23105–0.23153	170.4–184.3

Table 7. Same as in Table 6, for $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$

EFF scheme $\Delta\alpha_h^{(5)} = 0.02747$	range $\left(\sin^2\theta_{\rm eff}^{\rm lept}\right)_l$	range M_t [GeV]
80% C.L.	0.23119–0.23140	174.2–180.6
90% C.L.	0.23113–0.23146	172.2–182.7
95% C.L.	0.23107–0.23151	170.6–184.2

4 Discussion

As is well known, global analyses of the electroweak observables in the SM framework have frequently led to the derivation of preferred domains in the M_W, M_t plane at various C.L. [2, 12]. Although such studies are very valuable on general grounds, the focus of the present paper is quite different. Specifically, in Sect. 2 we have addressed the following question: given the present experimental values of M_W and M_t, and irrespective of other observables, what are the allowed ranges for these important parameters in the SM framework when the $(M_H)_{\rm L.B.}$ is taken into account? In Sect. 3 the same question is addressed in the case of $s^2_{\rm eff}$ and M_t. This approach conforms with the idea that, aside from the global fits, it is also important to compare the theory separately with the precise observables on which M_H depends most sensitively [13]. In fact, it is in principle possible that striking discrepancies of crucial observables and important information may be blurred in the global analysis. An obvious observation is that, if the SM is correct, the central values of M_W, M_t, and $(s^2_{\rm eff})_l$ must approach the allowed regions as the errors decrease, irrespective of what happens to other observables. We also note that in cases such as (M_W, M_t) and $\left((s^2_{\rm eff})_l, M_t\right)$, in which the central values lie well outside the allowed regions (cf. Figs. 1 and 3), the bounds derived in this approach are significantly more restrictive than those obtained in the indirect global analysis (cf. Fig. 16.2 of [2]). Finally, it is worth pointing out that analyses of the kind carried out in this paper are particularly simple. For instance, the ellipses in Figs. 1 and 3 are obtained from the experimental values by purely statistical means and the theory essentially enters in the derivation of the theoretical curves.

In order to implement this approach, in Sect. 2 we have compared the experimental values for M_W and M_t at various C.L. with the SM theoretical curves $M_W(M_H, M_t)$ for fixed M_H, imposing the restriction $M_H \geq 114.4\,$GeV. We have employed both the theoretical formulae of [1, 5] and considered two values of $\Delta\alpha_h^{(5)}$. As expected from the discussion in the Introduction, the M_W and M_t ranges are significantly reduced in the SM theoretical framework when the bound $M_H \geq 114.4\,$GeV is taken into account. Compatibility with the experimental 68% C.L. region only occurs in one of the alternatives we have considered and is at best marginal. In the experimental 80% C.L. domain, the current allowed M_W and M_t ranges vary from ($M_W = 80.402 \pm 0.020\,;\ M_t = 177.7 \pm 3.1$) GeV to ($M_W = 80.397 \pm 0.012\,;\ M_t = 178.3 \pm 2.0$) GeV, depending on the value of $\Delta\alpha_h^{(5)}$ and whether one employs the theoretical expressions of [1] or [5].

In order to belong to the allowed region, it is understood that pairs of M_W and M_t values from these intervals should be chosen so that they lie within the 80% C.L. domain. At the 90% and 95% C.L., the allowed M_W and M_t intervals are of course wider and can be read from Tables 1-4; however, to a very good approximation, in each table the mid-points are independent of the C.L.

The allowed M_W and M_t domains derived in this manner may be regarded as theoretically favored in the SM framework when the $(M_H)_{\rm L.B.}$ is taken into account, irrespective of other observables. Qualitatively, they indicate that compatibility with the theory would improve if $(M_W)^c_{\rm exp}$ would decrease and $(M_t)^c_{\rm exp}$ would increase. Assuming the validity of the SM, the central values of M_W and M_t must approach the allowed regions as the errors decrease. Of course, the precise end-point of this trajectory is not known, nor is it very clear what the optimal C.L. is to select the allowed region. On the other hand, the mid-points of the allowed regions provide natural representative examples. The fact that to very good approximation they are independent of the C.L. used in selecting the allowed regions (provided the C.L. are sufficiently large that there are allowed regions), makes them particularly attractive benchmarks. Therefore, we will consider, as an illustration of plausible future developments, a hypothetical, but representative scenario in which the experimental central points move to the mid-points of the current allowed intervals.

This would require a shift of -0.71 to $-0.85\sigma_{M_W}$ in $(M_W)^c_{\rm exp}$ and of $+0.67$ to $0.78\sigma_{M_t}$ in $(M_t)^c_{\rm exp}$. It is interesting to note that a change in $(M_W)^c_{\rm exp}$ of the same direction and magnitude occurred in the recent past: namely, the shift of $(M_W)^c_{\rm exp} = 80.451\,$GeV to $(M_W)^c_{\rm exp} = 80.426\,$GeV represented a $-0.76\sigma_{M_W}$ effect. Also, it is worthwhile to observe that the most precise M_W measurement, the LEP2 determination $(M_W)_{\rm LEP2} = 80.412 \pm 0.042\,$GeV [2] has a central value that is significantly closer than $(M_W)^c_{\rm exp}$ to the mid-points mentioned above. Finally, there is a very recent preliminary value $M_t = 180.1 \pm 5.4\,$GeV from the D0 collaboration that suggests that $(M_t)^c_{\rm exp}$ may significantly increase in the near future [14].

Assuming that the Higgs boson remains undiscovered, a natural question is: what would be the M_H prediction in this hypothetical scenario? We use $\sigma_{M_W} = 15\,$MeV and $\sigma_{M_t} = 2\,$GeV, which are projected for TeV-LHC [15], $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, and $\alpha_s(M_Z) = 0.118 \pm 0.002$. When the formulae of [1] are employed, the mid-points are (80.401, 177.9) GeV and we obtain the prediction

$$M_H = 114^{+46}_{-35}\,\text{GeV}; \quad M_H^{95} = 195\,\text{GeV}\,. \tag{2}$$

Instead, using [5], the mid-points are (80.397, 178.3) GeV, and this leads to

$$M_H = 114^{+47}_{-37}\,\text{GeV}; \quad M_H^{95} = 198\,\text{GeV}\,. \tag{3}$$

Equations (2) and (3) have been obtained without taking into account the $(M_H)_{\rm L.B.}$. Including its effect we find that M_H^{95} is shifted to 214 GeV in (2) and 218 GeV in (3).

In Sect. 3, we compared the experimental value for $s^2_{\rm eff}$ and M_t at various C.L. with the SM theoretical curves $s^2_{\rm eff} = s^2_{\rm eff}(M_H, M_t)$ for fixed M_H, taking into account the $(M_H)_{\rm L.B.}$ restriction. Here we considered two alternatives: the world average value for $s^2_{\rm eff}$ and the average $(s^2_{\rm eff})_{(l)}$ derived from the leptonic observables. In the first case, there is very good compatibility with $(M_H)_{\rm L.B.}$ and, in fact, the allowed $s^2_{\rm eff}$, M_t intervals are only reduced by relatively small amounts. In the second case, the 68% C.L. is forbidden by the $(M_H)_{\rm L.B.}$ and the allowed $(s^2_{\rm eff})_{(l)}$ and M_t intervals are significantly reduced. As in the case of the M_W, M_t analysis, we may consider a hypothetical scenario in which the experimental $(s^2_{\rm eff})_{(l)}$, M_t central values move in the future to representative points of the allowed region. In the $(s^2_{\rm eff})_{(l)}$, M_t case, it is convenient to use as benchmarks the mid-points of the ranges defined by the intersection of the B.C. with the C.L. ellipses (cf. Sect. 3). In order to illustrate how this shift in the central values would affect the M_H prediction, we assume again $\sigma_{M_t} = 2\,$GeV, an error $\sigma_{s^2_{{\rm eff},l}} = 0.00001$ for $(s^2_{\rm eff})_{(l)}$, as projected for $s^2_{\rm eff}$ in the GigaZ application of the NLC, and employ $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$, for which the mid-points discussed in Sect. 3 are (0.23133; 178.1 GeV). These inputs lead to

$$M_H = 115^{+37}_{-29}\,\text{GeV}; \quad M_H^{95} = 180\,\text{GeV}\,. \tag{4}$$

Including the effect of $(M_H)_{\rm L.B.}$, M_H^{95} in (4) is shifted to 196 GeV.

The central values in (2), (3) and (4) reflect the interesting feature that the mid-points are close to the $M_H = 114.4\,\mathrm{GeV}$ boundary curves.

The fact that the benchmark scenario we have considered involves a decrease in $(M_W)^c_{\rm exp}$ and an increase in $(M_t)^c_{\rm exp}$ and $(s^2_{\rm eff})^c_{(l)}$ can be readily understood qualitatively by observing the relative positions of the C.L. domains and the allowed theoretical curves in Figs. 1 and 3.

If instead we assume that the current central values for M_W, M_t, and $(s^2_{\rm eff})_{(l)}$ remain unaltered while the errors decrease to $\sigma_{M_W} = 15\,\mathrm{MeV}$, $\sigma_{M_t} = 2\,\mathrm{GeV}$, and $\sigma_{s^2_{{\rm eff},l}} = 0.00001$, the estimates of (2), (3) and (4) are replaced by

$$M_H = 45^{+25}_{-18}\,\mathrm{GeV};\quad M_H^{95} = 90\,\mathrm{GeV}\,, \tag{5}$$

$$M_H = 36^{+23}_{-17}\,\mathrm{GeV};\quad M_H^{95} = 79\,\mathrm{GeV}\,, \tag{6}$$

$$M_H = 59^{+21}_{-16}\,\mathrm{GeV};\quad M_H^{95} = 96\,\mathrm{GeV}\,, \tag{7}$$

respectively. Clearly, (5), (6) and (7) would indicate a sharp disagreement with the $(M_H)_{\rm L.B.}$!

An alternative possibility that would circumvent the incompatibility of (5), (6) and (7) would be an increase of $(M_t)^c_{\rm exp}$, with $(M_W)^c_{\rm exp}$ and $[(s^2_{\rm eff})_{(l)}]^c_{\rm exp}$ kept fixed. As pointed out in [14], such a shift would improve in general the compatibility with the SM. This can be readily understood from Figs. 1–3, since the C.L. ellipses would move towards the allowed region.

If $(M_t)^c_{\rm exp}$ increases by the current error 5.1 GeV, as discussed in [14], and we again employ $\sigma_{M_W} = 15\,\mathrm{MeV}$, $\sigma_{M_t} = 2\,\mathrm{GeV}$, we would obtain from the M_W input

$$M_H = 87^{+38}_{-29}\,\mathrm{GeV}\,;\qquad M_H^{95} = 155\,\mathrm{GeV}\,, \tag{8}$$

using [1], and

$$M_H = 74^{+36}_{-27}\,\mathrm{GeV}\,;\qquad M_H^{95} = 138\,\mathrm{GeV}\,, \tag{9}$$

from [5]. Employing the current value $(s^2_{\rm eff})_{(l)} = 0.23113$ and $\sigma_{s^2_{{\rm eff},l}} = 0.00001$, the result from the $(s^2_{\rm eff})_{(l)}$ input would be

$$M_H = 83^{+29}_{-22}\,\mathrm{GeV}\,;\qquad M_H^{95} = 134\,\mathrm{GeV}\,. \tag{10}$$

Unlike (5), (6) and (7) we note that (8), (9) and (10) are marginally compatible with the $(M_H)_{\rm L.B.}$. On the other hand, (8), (9) and (10), based on the $(M_t)^c_{\rm exp} = 179.4\,\mathrm{GeV}$ assumption, are significantly more restrictive than (2), (3) and (4) corresponding to the benchmark scenario discussed in this paper.

A qualitative difference between the two scenarios is that in (2), (3) and (4) the central experimental points reach the allowed region, while this does not happen in (8), (9) and (10). In fact, in order to reach the allowed region by varying $(M_t)^c_{\rm exp}$ alone, one would need a shift of +7.5 GeV or 1.5 times the current error if one employs the M_W input to predict M_H, and of 9.8 GeV or 1.9 times the current error if one uses $(s^2_{\rm eff})_{(l)}$.

Throughout this section we have employed $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$. The corresponding M_H estimates using the "theory driven" calculation $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$ are presented in Appendix B.

In summary, if the SM is correct, the experimental central values should approach the allowed region as the errors shrink, regardless of other observables, and on that basis the analysis of this paper suggests, as illustrated in Figs. 1 and 3, the possibility that $(M_W)^c_{\rm exp}$ will decrease, while $(M_t)^c_{\rm exp}$ and $(s^2_{\rm eff})^c_{(l)}$ will increase. In the hypothetical benchmark illustration that we have described, all these changes are $< 1\sigma$ in magnitude, so they are certainly not extreme. The fact that shifts of this magnitude have recently occurred gives some plausibility to this scenario. Thus, as the accuracy increases, it will be very interesting to see whether the central values approach the allowed regions preferred by the SM, remain where they are, or move in the opposite direction. In the last two cases a sharp disagreement with the SM would emerge, thus providing strong evidence for new physics!

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Appendix A

In the discussions of Sects. 2 and 3 we have kept $\Delta\alpha_h^{(5)}$ fixed at their central values. This has the advantage that the analysis is particularly simple and can be readily illustrated in terms of C.L. ellipses and the SM theoretical curves, as shown in the figures. If $\Delta\alpha_h^{(5)}$ is allowed to vary according to $\Delta\alpha_h^{(5)} = (\Delta\alpha_h^{(5)})^c \pm \sigma_{\Delta\alpha}$, the simplest procedure in the M_W, M_t case (Sect. 2) is to consider the χ^2 function:

$$\chi^2 = \frac{\left(M_W(M_H, M_t, \Delta\alpha_h^{(5)}) - M_W^c\right)^2}{\sigma^2_{M_W}} + \frac{(M_t - M_t^c)^2}{\sigma^2_{M_t}} + \frac{\left(\Delta\alpha_h^{(5)} - (\Delta\alpha_h^{(5)})^c\right)^2}{\sigma^2_{\Delta\alpha}}\,, \tag{A.1}$$

where M_W^c, M_t^c, and $(\Delta\alpha_h^{(5)})^c$ are the central values of $(M_W)_{\rm exp}$, $(M_t)_{\rm exp}$ and $\Delta\alpha_h^{(5)}$, respectively, and $M_W(M_H, M_t, \Delta\alpha_h^{(5)})$ is the SM theoretical curve that now depends on M_W, M_t and $\Delta\alpha_h^{(5)}$. For fixed χ^2 and M_H, (A.1) defines an implicit function $f(M_t, \Delta\alpha_h^{(5)}) = \chi^2$ relating M_t and $\Delta\alpha_h^{(5)}$. Varying $\Delta\alpha_h^{(5)}$ over an appropriate finite interval, one finds numerically the range spanned by M_t. Using $M_W(M_H, M_t, \Delta\alpha_h^{(5)})$ one then obtains the domain of variability of M_W. The resulting M_W, M_t ranges agree, up to at most minor changes, with those presented in the tables in Sect. 2.

In the $s^2_{\rm eff}$, M_t case, the effect of the $\Delta\alpha_h^{(5)} = (\Delta\alpha_h^{(5)})^c \pm \sigma_{\Delta\alpha}$ variation is more pronounced and, accordingly, we have derived the allowed intervals in the tables of Sect. 3 from a χ^2 analysis analogous to that explained above.

Appendix B

In this appendix we present the M_H estimates discussed in Sect. 4 when one employs the "theory driven" calculation $\Delta\alpha_h^{(5)} = 0.02747 \pm 0.00012$ [7], instead of $\Delta\alpha_h^{(5)} = 0.02761 \pm 0.00036$ [6].

We find that (2), (3) and (4), corresponding to our benchmark scenario with future projected errors, are replaced by

$$M_H = 115^{+43}_{-34}\,\text{GeV}; \quad M_H^{95} = 191\,\text{GeV}\,, \tag{B.1}$$

$$M_H = 114^{+45}_{-35}\,\text{GeV}; \quad M_H^{95} = 194\,\text{GeV}\,, \tag{B.2}$$

$$M_H = 115^{+19}_{-17}\,\text{GeV}; \quad M_H^{95} = 148\,\text{GeV}\,, \tag{B.3}$$

respectively.

Equations (5), (6) and (7), corresponding to the current central values with future projected errors, are replaced by

$$M_H = 48^{+25}_{-18}\,\text{GeV}; \quad M_H^{95} = 92\,\text{GeV}\,, \tag{B.4}$$

$$M_H = 38^{+23}_{-17}\,\text{GeV}; \quad M_H^{95} = 80\,\text{GeV}\,, \tag{B.5}$$

$$M_H = 65^{+12}_{-10}\,\text{GeV}; \quad M_H^{95} = 86\,\text{GeV}\,. \tag{B.6}$$

Finally, instead of (8), (9) and (10), corresponding to the $(M_t)^c_{\text{exp}} = (174.3 + 5.1)$ GeV assumption, with current $(M_W)^c_{\text{exp}}$ and $[(s^2_{\text{eff}})_{(l)}]^c_{\text{exp}}$ values, and future projected errors, we have

$$M_H = 91^{+38}_{-28}\,\text{GeV}; \quad M_H^{95} = 157\,\text{GeV}\,, \tag{B.7}$$

$$M_H = 78^{+35}_{-27}\,\text{GeV}; \quad M_H^{95} = 141\,\text{GeV}\,, \tag{B.8}$$

$$M_H = 92^{+16}_{-14}\,\text{GeV}; \quad M_H^{95} = 120\,\text{GeV}\,. \tag{B.9}$$

We see that the most significant change is between (4) and (B.3) derived from the $(s^2_{\text{eff}})_{(l)}$ input in the benchmark scenario discussed in this paper.

References

1. A. Ferroglia, G. Ossola, M. Passera, A. Sirlin, Phys. Rev. D **65**, 113002 (2002) [hep-ph/0203224]
2. LEP Collaboration, hep-ex/0312023
3. A. Sirlin, Phys. Rev. D **22**, 971 (1980); D **29**, 89 (1984); W.J. Marciano, A. Sirlin, Phys. Rev. D **22**, 2695 (1980)
4. M. Awramik, M. Czakon, Phys. Rev. Lett. **89**, 241801 (2002) [hep-ph/0208113]; A. Onishchenko, O. Veretin, Phys. Lett. B **551**, 111 (2003) [hep-ph/0209010]; M. Awramik, M. Czakon, A. Onishchenko, O. Veretin, Phys. Rev. D **68**, 053004 (2003) [hep-ph/0209084]; M. Awramik, M. Czakon, Phys. Lett. B **568**, 48 (2003) [hep-ph/0305248]
5. M. Awramik, M. Czakon, A. Freitas, G. Weiglein, hep-ph/0311148
6. H. Burkhardt, B. Pietrzyk, LAPP-EXP-2001-03
7. J.F. de Troconiz, F.J. Yndurain, Phys. Rev. D **65**, 093002 (2002) [hep-ph/0107318]
8. A. Ferroglia, G. Ossola, A. Sirlin, Phys. Lett. B **507**, 147 (2001) [hep-ph/0103001]; hep-ph/0106094; A. Sirlin, Nucl. Phys. Proc. Suppl. **116**, 53 (2003) [hep-ph/0210361]
9. K. Aoki et al., Prog. Theor. Phys. Suppl. **73**, 1 (1982); M. Böhm, H. Spiesberger, W. Hollik, Fortsch. Phys. **34**, 687 (1986); W. Hollik, Fortsch. Phys. **38**, 165 (1990); E. Kraus, Annals Phys. **262**, 155 (1998) [hep-th/9709154]; P.A. Grassi, Nucl. Phys. B **560**, 499 (1999) [hep-th/9908188]
10. B.A. Kniehl, A. Sirlin, Eur. Phys. J. C **16**, 635 (2000) [hep-ph/9907293]
11. A. Sirlin, in Proceedings of the 19th International Symp. on Photon and Lepton Interactions at High Energy LP99, edited by J.A. Jaros, M.E. Peskin, Int. J. Mod. Phys. A **15S1**, 398 (2000) [eConf C990809 (2000) 398] [hep-ph/9912227]; A. Sirlin, J. Phys. G **29**, 213 (2003) [hep-ph/0209079]
12. J. Erler, AIP Conf. Proc. **670**, 227 (2003) [hep-ph/0212272]
13. G. Degrassi, P. Gambino, M. Passera, A. Sirlin, Phys. Lett. B **418**, 209 (1998) [hep-ph/9708311]; M.S. Chanowitz, Phys. Rev. Lett. **80**, 2521 (1998) [hep-ph/9710308]; W.J. Marciano, J. Phys. G **29**, 225 (2003)
14. P. Gambino, Int. J. Mod. Phys. A **19**, 808 (2004) [arXiv:hep-ph/0311257]
15. W. Hollik, J. Phys. G **29**, 131 (2003)